LECTURES NOTES

ON

MACHINE DESIGN II

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Module I: Theories of Failure

THEORIES OF FAILURE UNDER STATIC LOADING

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uniaxial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi- axial stress are as follows:

- Maximum principal (or normal) stress theory (also known as Rankine's theory).
- Maximum shear stress theory (also known as Guest's or Tresca's theory).
- Maximum principal (or normal) strain theory (also known as Saint Venant theory).
- Maximum strain energy theory (also known as Haigh's theory).
- Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

1. Maximum principal stress theory (Rankine's theory)

According this theory failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test. Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (F.S.) into consideration, the maximum principal or normal stress σ_{t1} in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}$$

where σ_{yt} = Yield point stress in tension as determined from simple tension test, and

σ_u = Ultimate stress.

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

2. <u>Maximum shear stress theory (Guest's or Tresca's theory)</u>

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt}/F.S. \qquad ...(i)$$

where τ_{max} = Maximum shear stress in a bi-axial stress system,

τ_{yt} = Shear stress at yield point as determined from simple tension test, and

$F.S.$ = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation (*i*) may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is mostly used for designing members of ductile materials**.**

3. <u>Maximum principal strain theory (Saint Venant theory)</u>

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (i.e. strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E}$$

$\therefore$ According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.S.} \qquad ...(i)$$

where σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,

ε = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

E = Young's modulus, and

$F.S.$ = Factor of safety.

From equation (*i*), we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

4. <u>Maximum strain energy theory (Haigh's theory)</u>

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (i.e. strain energy at the yield point) per unit volume as determined from simple tension test.

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \quad \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

5. <u>Maximum distortion energy theory (also known as Hencky and Von Mises theory)</u>

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (i.e. distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

Example 1:

The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to 1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory. Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.

Given : P_{t1} = 10 kN ; P_s = 5 kN ; $\sigma_{t(el)}$ = 100 MPa = 100 N/mm^2 ; $1/m$ = 0.3

Let d = Diameter of the bolt in mm.

$\therefore$ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\ d^2\ \text{mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\ d^2} = \frac{12.73}{d^2}\ \text{kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\ d^2} = \frac{6.365}{d^2}\ \text{kN/mm}^2$$

1. *According to maximum principal stress theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4+4}\right] = \frac{15.365}{d^2}\ \text{kN/mm}^2 = \frac{15\,365}{d^2}\ \text{N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

$\therefore$ $d^2 = 15\,365/100 = 153.65$ or $d = 12.4$ mm **Ans.**

2. *According to maximum shear stress theory*

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{9}{d^2}\ \text{kN/mm}^2 = \frac{9000}{d^2}\ \text{N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

$\therefore$ $d^2 = 9000 / 50 = 180$ or $d = 13.42$ mm **Ans.**

3. *According to maximum principal strain theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-\,2.635}{d^2}\ \text{kN/mm}^2$$

$$= \frac{-\,2635}{d^2}\ \text{N/mm}^2$$

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$$d^2 = 16\,156 / 100 = 161.56 \quad \text{or} \quad d = 12.7\ \text{mm}$$ **Ans.**

4. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-\,2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$$\therefore \quad d^4 = 26\,724 \quad \text{or} \quad d = 12.78\ \text{mm}$$ **Ans.**

5. According to maximum distortion energy theory

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\,391}{d^4} = 1$$

$$\therefore \quad d^4 = 32\,391 \text{ or } d = 13.4 \text{ mm } \textbf{Ans.}$$

Example 2:

A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a torque T. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. The maximum shear stress; and 3. the maximum distortion strain energy theory of yielding.

Given: $d = 50$ mm ; $M = 2000$ N-m $= 2 \times 10^6$ N-mm ; $\sigma_{yt} = 200$ MPa $= 200$ N/mm^2

1. According to maximum principal stress theory

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3 = 12\,273 \text{ mm}^3$$

$\therefore$ Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2 \times 10^6}{12\,273} = 163 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16\,T}{\pi\,(50)^3} = 0.0407 \times 10^{-3}\,T \text{ N/mm}^2$$

$$\ldots \left[\because T = \frac{\pi}{16} \times \tau \times d^3\right]$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} + \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

Minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} - \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \text{...(Taking } F.S. = 1)$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \text{...(Taking } F.S. = 1)$$

$$\therefore \quad 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = 200$$

$$6642.5 + 1.65 + 10^{-9}\, T^2 = (200 - 81.5)^2 = 14\,042$$

$$T^2 = \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9$$

or $T = 2118 \times 10^3$ N-mm $= 2118$ N-m **Ans.**

2. *According to maximum shear stress theory*

We know that according to maximum shear stress theory,

$$\tau_{max} = \tau_{yt} = \frac{\sigma_{yt}}{2}$$

$$\therefore \quad \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = \frac{200}{2} = 100$$

$$6642.5 + 1.65 \times 10^{-9}\, T^2 = (100)^2 = 10\,000$$

$$T^2 = \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9$$

$\therefore$ $T = 1426 \times 10^3$ N-mm $= 1426$ N-m **Ans.**

3. *According to maximum distortion strain energy theory*

We know that according to maximum distortion strain energy theory

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = (\sigma_{yt})^2$$

$$\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]^2$$

$$- \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]\left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right] = (200)^2$$

$$2\left[(81.5)^2 + 6642.5 + 1.65 \times 10^{-9}\, T^2\right] - \left[(81.5)^2 - 6642.5 + 1.65 \times 10^{-9}\, T^2\right] = (200)^2$$

$$(81.5)^2 + 3 \times 6642.5 + 3 \times 1.65 \times 10^{-9}\, T^2 = (200)^2$$

$$26\ 570 + 4.95 \times 10^{-9}\, T^2 = 40\ 000$$

$$T^2 = \frac{40\ 000 - 26\ 570}{4.95 \times 10^{-9}} = 2713 \times 10^9$$

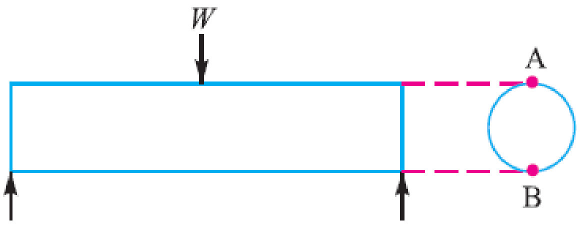
$$\therefore \quad T = 1647 \times 10^3 \text{ N-mm} = 1647 \text{ N-m} \quad \textbf{Ans.}$$

STRESS DUE TO VARIABLE LOADING CONDITIONS

A few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads).

Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load W, as shown in the figure. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (i.e. at point A) are under compressive stress and the lower fibres (i.e. at point B) are under tensile stress. After half a revolution, the point B occupies the position of point A and the point A occupies the position of point B. Thus the point B is now under compressive stress and the point A under tensile stress. The speed of variation of these stresses depends upon the speed of the beam. From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or vice versa, are known as completely reversed or cyclic stresses.



Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals. In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 2, is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 3. A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig. 4. A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 4, the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as endurance or fatigue limit (σe). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term endurance strength may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions. We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 3. In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress verses time diagram for fluctuating stress having values σmin and σmax is shown in Fig. 5. The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σv. The following relations are derived from Fig. 5:

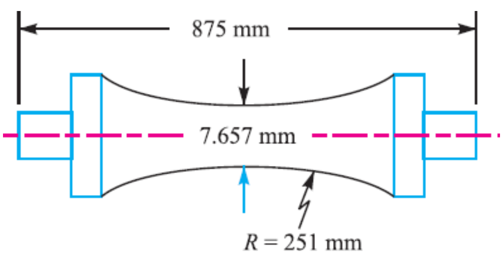


Fig. 2 standard specimen

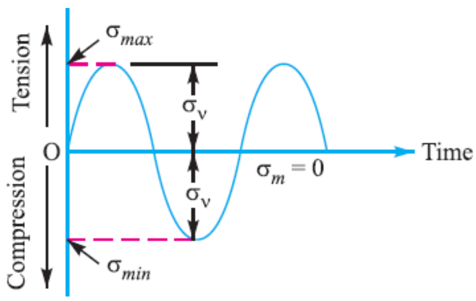


Fig. 3 completely reversed stress

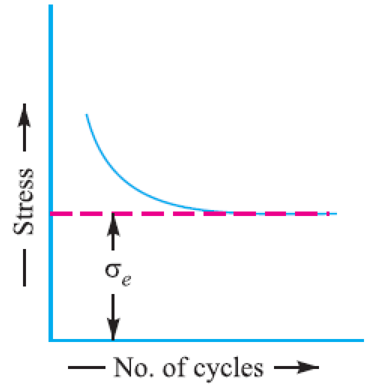


Fig. 4 Endurance/fatigue limit

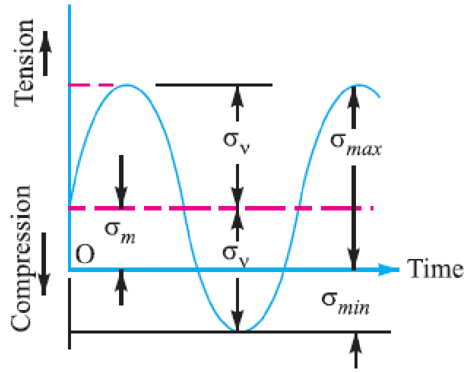


Fig. 5 fluctuating stress

Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

EFFECT OF MISCELLANEOUS FACTORS ON ENDURANCE LIMIT

Corrected endurance limit for variable bending load $\boxed{\sigma_e' = \sigma_e . K_b . K_{sr} . K_{sz}}$ (Page 4.14, Design data handbook by Jalaludeen)

Corrected endurance limit for variable axial load $\boxed{\sigma_e' = \sigma_e . K_d . K_{sr} . K_{sz}}$ (Page 4.14, Design data handbook by Jalaludeen)

Corrected endurance limit for variable torsional load $\boxed{\sigma_e' = \sigma_e . K_s . K_{sr} . K_{sz}}$ (Page 4.14, Design data handbook by Jalaludeen)

Where,

K_a = load correction factor for revered axial load

K_b = load correction factor for revered bending load

K_{sr} = surface finish factor

K_{sz} = size factor

σ_e = endurance limit/fatigue stress

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for faliure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

And $\sigma_e \approx 0.5\sigma_u$
$\approx 0.85\sigma_y$ (for steel) (Page 4.14, Design data handbook by Jalaludeen)

Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration. It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Figure 6. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

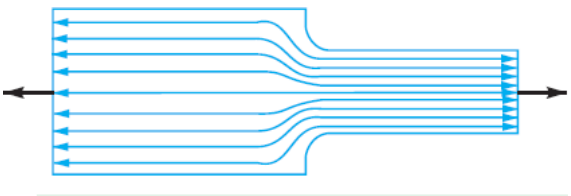


Figure 6 stress concentration

Theoretical Stress Concentration Factor (K_t)

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

The value of K_t depends upon the material and geometry of the part. (Table 4.9 to 4.16, Design data handbook by Jalaludeen).

Methods of Reducing Stress Concentration

1. By providing fillets as shown in Fig. 7

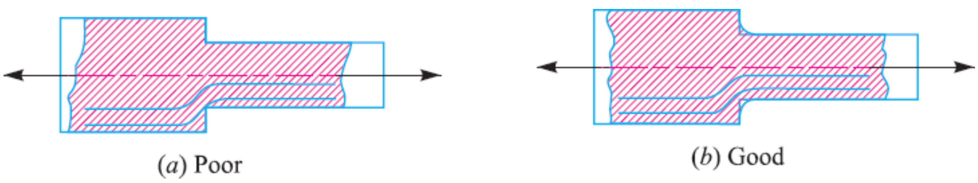


Figure 7 Fillets to improve stress concentration

2. By providing notches shown in Fig. 8

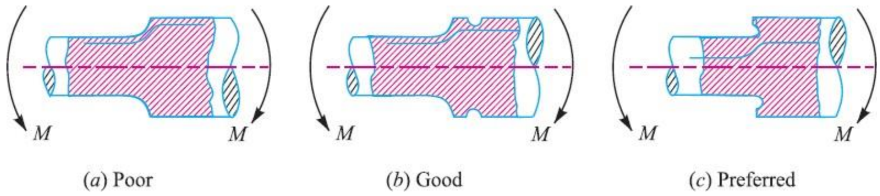


Figure 8 Notches to improve stress concentration

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term notch sensitivity is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

or $$K_f = 1 + q\,(K_t - 1) \qquad \text{...[For tensile or bending stress]}$$

and $$K_{fs} = 1 + q\,(K_{ts} - 1) \qquad \text{...[For shear stress]}$$

(Page 4.14, Design data handbook by Jalaludeen)

Where,

K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Combined Steady and Variable Stress

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. 9 as functions of variable stress (σ_v) and mean stress (σ_m).

The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

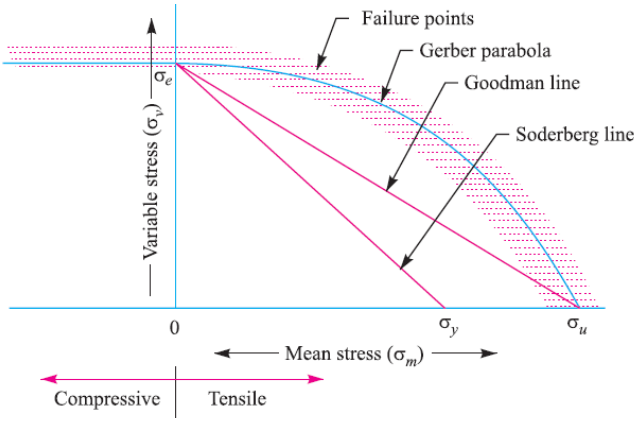


Figure 9 Combined mean and variable stress

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

1. Goodman method, and 2. Soderberg method.

Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σu), as shown by line AB in Fig. 10, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

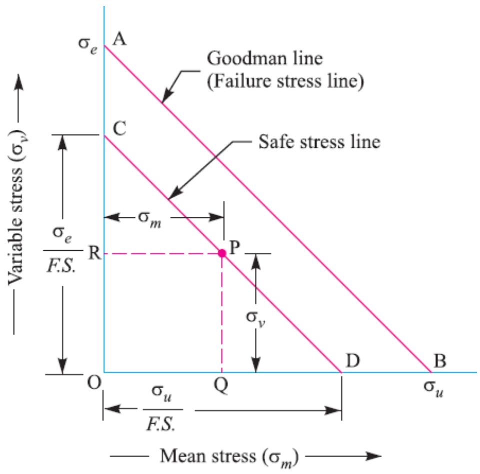


Figure 10 Goodman method

σ_u is called Goodman's failure stress line. If a suitable factor of safety (F.S.) is applied to endurance limit and ultimate strength, a safe stress line *CD* may be drawn parallel to the line

AB. Let us consider a design point P on the line *CD*. Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad ...(\because\ QD = OD - OQ)$$

$$\therefore \qquad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads. Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (i) must be altered to include this effect. In such cases, the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v). The equation (i) may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

where $F.S.$ = Factor of safety,
σ_m = Mean stress,
σ_u = Ultimate stress,
σ_v = Variable stress,
σ_e = Endurance limit for reversed loading, and
K_f = Fatigue stress concentration factor.

(Equation 4.50, pp. 4.14, Design data handbook by Jalaludeen)

Considering the load factor, surface finish factor and size factor, the equation (ii) may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad ...(\because\ \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

where K_b = Load factor for reversed bending load,
K_{sur} = Surface finish factor, and
K_{sz} = Size factor.

Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line *AB* in Fig. 11, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength. the line AB connecting σe and σy, as shown in Fig. 11, is called Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point P on the line *CD*. Now from similar triangles *COD* and *PQD*,

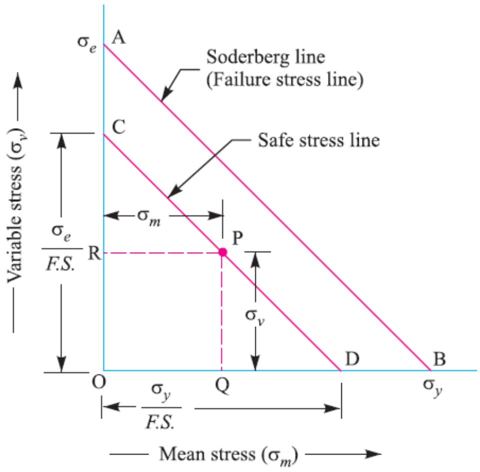


Figure 11 Soderberg method

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD}$$

$$\ldots(\because\ QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

(Equation 4.51, pp. 4.14, Design data handbook by Jalaludeen)

Module II: Design of IC Engine Components

DESIGN OF CYLINDER

Assumptions:

The piston side thrust tends to bend the cylinder wall, but the stress in the wall due to side thrust is very small and hence it may be neglected.

Let

D_0 = Outside diameter of the cylinder in mm,
D = Inside diameter of the cylinder in mm,
p = Maximum pressure inside the engine cylinder in N/mm^2,
t = Thickness of the cylinder wall in mm, and
$1/m$ = Poisson's ratio. It is usually taken as 0.25.

The apparent longitudinal stress is given by

$$\sigma_l = \frac{\text{Force}}{\text{Area}} = \frac{\frac{\pi}{4} \times D^2 \times p}{\frac{\pi}{4}[(D_0)^2 - D^2]} = \frac{D^2 \,.\, p}{(D_0)^2 - D^2}$$

and the apparent circumferential stresss is given by

$$\sigma_c = \frac{\text{Force}}{\text{Area}} = \frac{D \times l \times p}{2t \times l} = \frac{D \times p}{2t}$$

(where l is the length of the cylinder and area is the projected area)

Net longitudinal stress $= \sigma_l - \frac{\sigma_c}{m}$

net circumferential stress $= \sigma_c - \frac{\sigma_l}{m}$

The thickness of a cylinder wall (t) $= \frac{p \times D}{2\sigma_c} + C$

where p = Maximum pressure inside the cylinder in N/mm^2,

D = Inside diameter of the cylinder or cylinder bore in mm,

σ_c = Permissible circumferential or hoop stress for the cylinder material

C = Allowance for reboring. (its value varies from 6 to 12)

Thickness of the water jacket wall $= 0.032\, D + 1.6$ mm

(Eq. 15.6, pp. 15.5, design handbook by Jalaludeen)

DESIGN OF CONNECTING ROD

In designing a connecting rod, the following dimensions are required to be determined :

1. Dimensions of cross-section of the connecting rod,
2. Dimensions of the crankpin at the big end and the piston pin at the small end,
3. Size of bolts for securing the big end cap, and
4. Thickness of the big end cap.

2. Dimensions of crankpin at the big end and the piston pin at the small end

We have the maximum gas pressure $\boxed{F_g = \frac{\pi}{4} D^2 p}$ (Eq. 16.1, pp. 16.4, Jalaludeen)

Now diameter of piston pin $\boxed{d_1 = \frac{F_g}{p_{b1} \times l_1}}$ (Eq. 16.12, pp. 16.5, Jalaludeen)

Where d_1 = diameter of the piston pin

p_{b1} = design bearing pressure for small end in N/mm^2

l_1 = length of piston pin in mm = (1.5 to 2) d_1 (Eq. 16.14, pp. 16.6, Jalaludeen)

Diameter of the crank pin $\boxed{d_2 = \frac{F_g}{p_{b2} \times l_2}}$ (Eq. 16.13, pp. 16.6, Jalaludeen)

Where d_2 = diameter of the crank pin

p_{b2} = design bearing pressure for big end in N/mm^2

l_2 = length of piston pin in mm = (1.0 to 1.25) d_2 (Eq. 16.15, pp. 16.6, Jalaludeen)

3. Size of Bolts for securing big end cap

Maximum inertia pressure acting on the bolts $\boxed{F_{im} = \frac{W_r}{g} \times \frac{\omega^2 r}{1000}\left[1 + \frac{1}{l/r}\right]}$

(Eq. 16.10, pp. 16.5, Jalaludeen)

Where W_r = weight of the reciprocating parts in N

ω = angular speed of the crank in rad/s

r = radius of crank shaft in mm

So core dia. of the bolt $\boxed{d_c = \left[\frac{2F_{im}}{\pi\sigma_t}\right]^{1/2}}$ (Eq. 16.11, pp. 16.5, Jalaludeen)

Nominal dia. of bolt $\boxed{d_b = 1.2 d_c}$ (Eq. 16.11, pp. 16.5, Jalaludeen)

Where σ_t = allowable tensile stress on the piston rod or on the bolt

$= \boxed{\sigma_y / FS}$ = yield stress/ factor of safety (pp. 16.5, Jalaludeen)

No of bolts = 02 (pp. 16.5, Jalaludeen)

4. Thickness of big-end cap

Thickness of big end cap $\boxed{t_c = \left[\dfrac{F_{im} \times l'}{b.\sigma_{bc}} \right]^{1/2}}$ (Eq. 16.21, pp. 16.6, Jalaludeen)

Where l'= distance between the bolt centers at the big end in mm

$= \boxed{d_2 + 2t_b + d_b + 2t_m}$ (pp. 16.6, Jalaludeen)

d_2 = dia. of the crank pin in mm

t_b = thickness of bush in mm

d_b = nominal dia. of the bolt in mm.

t_m = marginal thickness in mm

b = width of cap in the big end of the connecting rod in mm.

$= l_2 - 2t_b$ (pp. 16.6, Jalaludeen)

σ_{bc} = allowable bending stress in the cap of the big end in N/mm^2

l_2 = length of piston pin in mm

DESIGN OF FLYWHEEL

INTRODUCTION

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.

FLYWHEEL VS GOVERNOR

Flywheel	Governor
In machines where the operation is intermittent like punching machines, shearing machines, riveting machines, crushers etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.	The function of a governor in engine is entirely different from that of a flywheel. It regulates the mean speed of an engine when there are variations in the load, e.g. when the load on the engine increases, it becomes necessary to increase the supply of working fluid. On the other hand, when the load decreases, less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load condition and keeps the mean speed within certain limits.

COEFFICIENT OF FLUCTUATION OF SPEED

Coefficient of fluctuation of speed $C_S = \frac{N_2 - N_1}{N}$ **(Eq. 18.1, pp.18.3, Jalaludeen)**

$$= \frac{v_2 - v_1}{v}$$

$$= \frac{\omega_2 - \omega_1}{\omega}$$

Where, N_1 = Minimum speed of the flywheel in rpm

N_2 = Maximum speed of the flywheel in rpm

N = Average speed of the flywheel in rpm

v_1, v_2= Min. and Max. velocity of flywheel in m/s, respectivly

v = velocity of flywheel in m/s

ω = angular velocity of flywheel in rad/s

ω_1, ω_2 = min. and max. angular velocities of flywheel in rad/s, respectively

The value of C_s can obtained from Table 18.1, pp. 18.6, Jalaludeen, depending upon type of engine used

FLUCTUATION OF ENERGY

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider a turning moment diagram for a single cylinder double acting steam engine as shown in Fig. 1. The vertical ordinate represents the turning moment and the horizontal ordinate (abscissa) represents the crank angle. A little consideration will show that the turning moment is zero when the crank angle is zero. It rises to a maximum value when crank angle reaches 90º and it is again zero when crank angle is 180º. This is shown by the curve abc in Fig. 1 and it represents the turning moment diagram for outstroke. The curve cde is the turning moment diagram for instroke and is somewhat similar to the curve abc. Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line AF. The height of the ordinate aA represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aAFe* is proportional to the work done against the mean resisting torque.

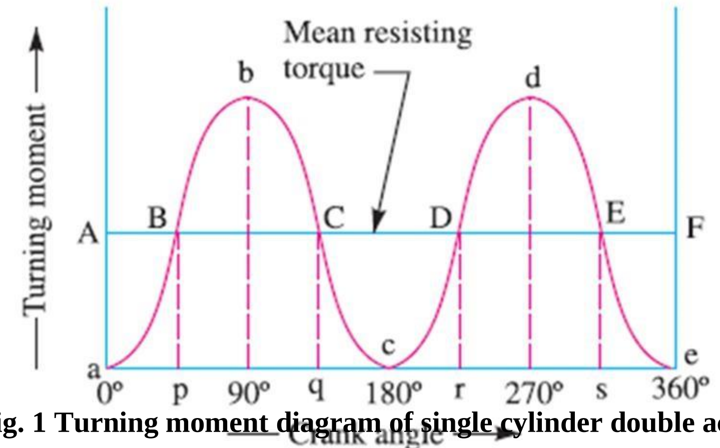


Fig. 1 Turning moment diagram of single cylinder double acting steam engine

It can be seen in Figure, that the mean resisting torque line AF cuts the turning moment diagram at points B, C, D and E. When the crank moves from ‘a’ to ‘p’ the work done by the engine is equal to the area aBp, whereas the energy required is represented by the area aABp. In other words, the engine has done less work (equal to the area aAB) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from p to q, the work done by the engine is equal to the area pBbCq, whereas the requirement of energy is represented by the area pBCq. Therefore the engine has done more work than the requirement. This excess work (equal to the area BbC) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from p to q. Similarly when the crank moves from q to r, more work is taken from the engine than is developed. This loss of work is represented by the area CcD. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from q to r. As the crank moves from r to s, excess energy is again developed given by the area DdE and the speed again increases. As the piston moves from s to e, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called **fluctuation of energy**. The areas BbC, CcD, DdE etc. represent fluctuations of energy. A turning moment diagram for a four stroke internal combustion engine is shown in Figure 2

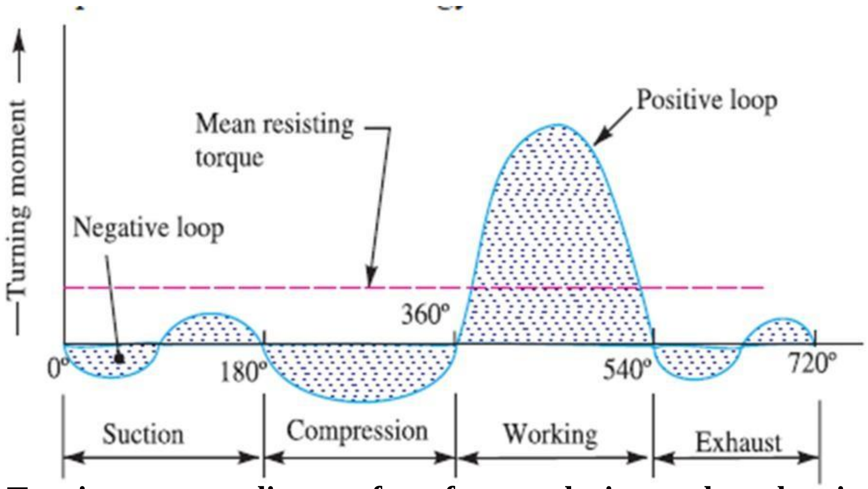


Fig. 2 Turning moment diagram for a four stroke internal combustion engine

MAXIMUM FLUCTUATION OF ENERGY

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 3. The horizontal line AG represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

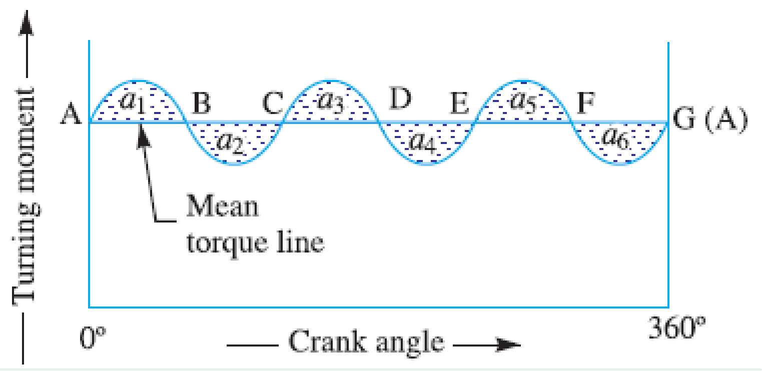


Fig. 3 Turning moment diagram for a multi-cylinder engine

Let the energy in the flywheel at A = E,

Energy at B = E + a_1

Energy at C = E + $a_1 - a_2$

Energy at D = E + $a_1 - a_2 + a_3$

Energy at E = E + $a_1 - a_2 + a_3 - a_4$

Energy at F = E + $a_1 - a_2 + a_3 - a_4 + a_5$

Energy at G = E + $a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ = Energy at A

Let us now suppose that the maximum of these energies is at B and minimum at E.

∴ Maximum energy in the flywheel

= E + a_1

and minimum energy in the flywheel

= E + $a_1 - a_2 + a_3 - a_4$

∴ Maximum fluctuation of energy,

ΔE = Maximum energy – Minimum energy

= (E + a_1) – (E + $a_1 - a_2 + a_3 - a_4$) = $a_2 - a_3 + a_4$

COEFFICIENT OF FLUCTUATION OF ENERGY

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by C_e. Mathematically, coefficient of fluctuation of energy,

C_e = (Maximum fluctuation of energy/Work done per cycle)

So $C_e = \dfrac{\Delta E}{\left(\dfrac{60P}{N}\right)}$ **(Eq. 18.5, pp.18.4, Jalaludeen) (For 2-S engine)**

$C_e = \dfrac{\Delta E}{\left(\dfrac{120P}{N}\right)}$ **(Eq. 18.5, pp.18.4, Jalaludeen) (For 4-S engine)**

The value of C_e can obtained from Table 18.2, pp. 18.6, Jalaludeen, depending upon type of engine used

ENERGY STORED IN A FLYWHEEL

A flywheel is shown in Fig. 4. It has already been discussed that when a flywheel absorbs energy its speed increases and when it gives up energy its speed decreases.

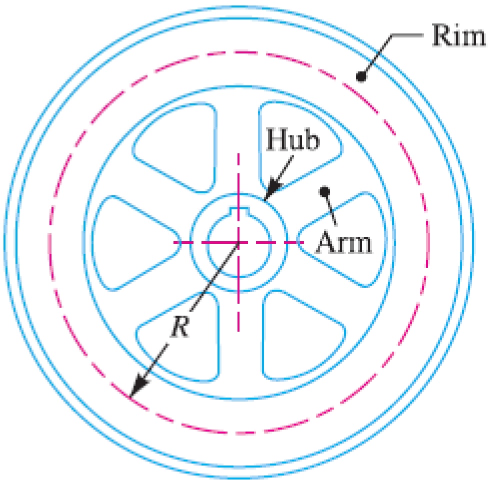


Fig. 4 Flywheel

Let

m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about the axis of rotation in kg-m^2 = $m.k^2$,

N_2 and N_1 = Maximum and minimum speeds during the cycle in r.p.m., respectively

ω_2 and ω_1 = Maximum and minimum angular speeds during the cycle in rad / s, respectively

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_2 + N_1}{2}$

ω = Mean angular speed during the cycle in rad / s $= \dfrac{\omega_2 + \omega_1}{2}$

C_S = Coefficient of fluctuation of speed $= \dfrac{N_2 - N_1}{N} = \dfrac{\omega_2 - \omega_1}{\omega}$

We know that mean kinetic energy of the flywheel, $E = \dfrac{1}{2} I\omega^2$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

ΔE = Maximum K.E. — Minimum K.E.

$$
\begin{aligned}
&= \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2 = \frac{1}{2} I\left(\omega_2^2 - \omega_1^2\right) \\
&= \frac{1}{2} I\,(\omega_2 + \omega_1)(\omega_2 - \omega_1) \\
&= I.\omega.(\omega_2 - \omega_1) \\
&= I.\omega^2 \frac{(\omega_2 - \omega_1)}{\omega} \\
&= I.\omega^2.C_S \\
&= m.k^2.\omega^2.C_S \\
&= 2.E.C_S
\end{aligned}
$$

(Eq. 18.1, pp.18.3, Jalaludeen)

$\because \omega = \dfrac{\omega_2 + \omega_2}{2}$

And $I = m.k^2$, $E = \dfrac{1}{2} I.\omega^2$

And $\boxed{k^2 = \frac{D_o^2 + (D_o - 2t)^2}{8}}$ (for rim flywheel), **(pp.18.3, Jalaludeen)**

$\boxed{k^2 = \frac{D_o^2}{4} = \frac{D^2}{4} = R^2}$ (for rim flywheel, when t is very small), **(pp.18.3, Jalaludeen)**

$\boxed{k^2 = \frac{D_o^2}{8}}$ (for disc flywheel, as D_i is zero), **(pp.18.3, Jalaludeen)**

Where, D_o = Outer diameter of rim in m.

D_i= inner diameter of rim in m.

R = mean radius of rim in m.

From this expression (Eq. 18.3), the mass of the flywheel rim may be determined.

Notes: In the above expression, only the mass moment of inertia of the rim is considered and the mass moment of inertia of the hub and arms is neglected. This is due to the fact that the major portion of weight of the flywheel is in the rim and a small portion is in the hub and arms. Also the hub and arms are nearer to the axis of rotation, therefore the moment of inertia of the hub and arms is very small.

$\boxed{m = \text{Volume} \times \text{Density} = 2\,\pi\, R \times A \times \rho}$ **(Eq. 18.2, pp.18.3, Jalaludeen)**

From this expression, the value of the cross-sectional area of the rim can be obtained.

Assuming the cross-section of the rim to be rectangular, then

$A = b \times t$

where b = Width of the rim, and

t = Thickness of the rim

b / t=0.65 to 2 **(pp.18.3, Jalaludeen)**

Question1 : The turning moment diagram for a petrol engine is drawn to the following scales: Turning moment, 1 mm = 5 N-m; Crank angle, 1 mm = 1º. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line, taken in order are 295, 685, 40, 340, 960, 270 mm². Determine the mass of 300 mm diameter flywheel rim when the coefficient of fluctuation of speed is 0.3% and the engine runs at 1800 r.p.m. Also determine the cross-section of the rim when the width of the rim is twice of thickness. Assume density of rim material as 7250 kg / m³.

Solution. Given : D = 300 mm or R = 150 mm = 0.15 m ; C_S = 0.3% = 0.003 ; N = 1800 r.p.m. or $\omega = 2\,\pi \times 1800 / 60 = 188.5$ rad/s ; $\rho = 7250$ kg / m³

Mass of the flywheel

Let m = Mass of the flywheel in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 22.6.

Since the scale of turning moment is 1 mm = 5 N-m, and scale of the crank angle is 1 mm = 1° = π / 180 rad, therefore 1 mm² on the turning moment diagram.

$$= 5 \times \pi / 180 = 0.087 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.6, we find that

Energy at $B = E + 295$

Energy at $C = E + 295 - 685 = E - 390$

Energy at $D = E - 390 + 40 = E - 350$

Energy at $E = E - 350 - 340 = E - 690$

Energy at $F = E - 690 + 960 = E + 270$

Energy at $G = E + 270 - 270 = E$ = Energy at A

From above we see that the energy is maximum at B and minimum at E.

∴ Maximum energy $= E + 295$

and minimum energy $= E - 690$

ΔE = Maximum energy — Minimum energy

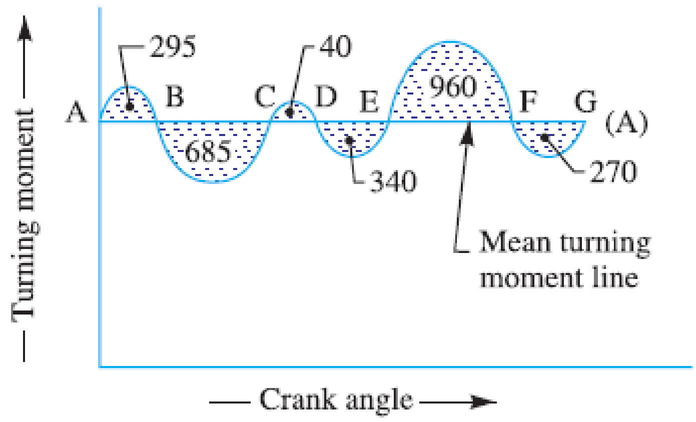
$= (E + 295) - (E - 690) = 985 \text{ mm}^2$

$= 985 \times 0.087 = 86$ N-m

So,

$$86 = m.R^2.\omega^2.C_S = m\,(0.15)^2\,(188.5)^2\,(0.003) = 2.4\,m$$

$$m = 86 / 2.4 = 35.8 \text{ kg}$$



∴ Cross-sectional area of rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$35.8 = A \times 2\pi R \times \rho = 2t^2 \times 2\pi \times 0.15 \times 7250 = 13\,668\,t^2$$

$$\therefore \quad t^2 = 35.8 / 13\,668 = 0.0026 \quad \text{or} \quad t = 0.051 \text{ m} = 51 \text{ mm}$$

and

$$b = 2\,t = 2 \times 51 = 102 \text{ mm}$$

Assignment:

Q1. The intercepted areas between the output torque curve and the mean resistance line of a turning moment diagram for a multicylinder engine, taken in order from one end are as follows: – 35, + 410, – 285, + 325, – 335, + 260, – 365, + 285, – 260 mm^2. The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speed is 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed. Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The density of the material of the flywheel may be taken as 7200 kg / m^3. The rim is rectangular with the width 2 times the thickness. Neglect effect of arms, etc.

Q2. A single cylinder double acting steam engine develops 150 kW at a mean speed of 80 r.p.m. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is ± 2% of mean speed. If the mean diameter of the flywheel rim is 2 metres and the hub and spokes provide 5 % of the rotational inertia of the wheel, find the mass of the flywheel and cross-sectional area of the rim. Assume the density of the flywheel material (which is cast iron) as 7200 kg / m^3.

Q3. A single cylinder, single acting, four stroke oil engine develops 20 kW at 300 r.p.m. The workdone by the gases during the expansion stroke is 2.3 times the workdone on the gases during the compression and the workdone during the suction and exhaust strokes is negligible. The speed is to be maintained within ± 1%. Determine the mass moment of inertia of the flywheel.

STRESSES IN A FLYWHEEL RIM

The following types of stresses are induced in the rim of a flywheel:

1. Tensile stress due to centrifugal force,
2. Tensile bending stress caused by the restraint of the arms, and
3. The shrinkage stresses due to unequal rate of cooling of casting.

These stresses may be very high but there is no easy method of determining. This stress is taken care of by a factor of safety.

1. Tensile stress due to the centrifugal force

Let

b = Width of rim,

t = Thickness of rim,

A = Cross-sectional area of rim = b × t,

D = Mean diameter of flywheel

R = Mean radius of flywheel,

ρ = Density of flywheel material,

ω = Angular speed of flywheel,

v = Linear velocity of flywheel, and

σ_t = Tensile or hoop stress.

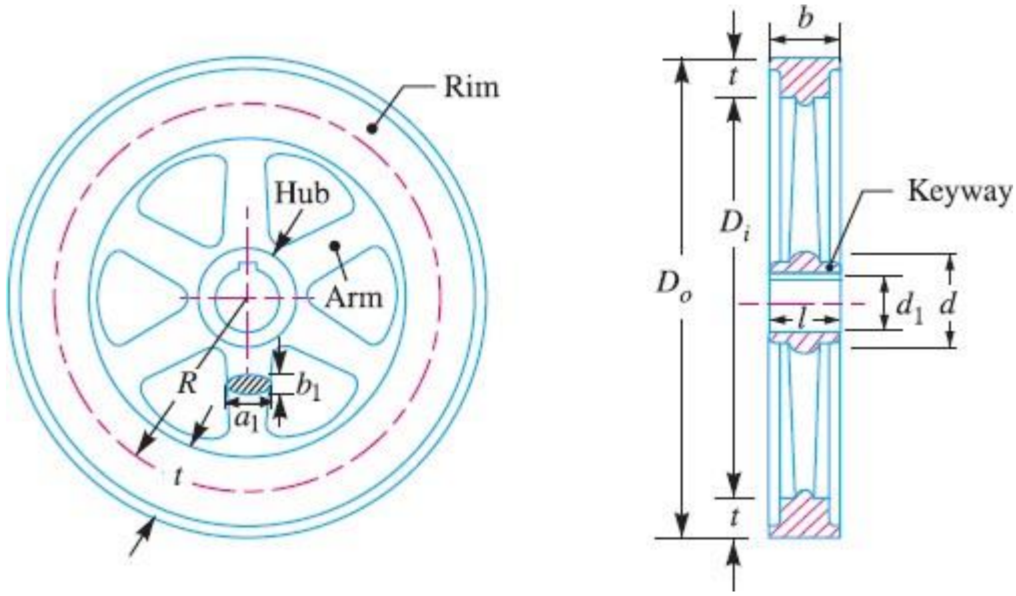


Fig. 5 Different dimensions of flywheel

Total vertical bursting force across the rim diameter X-Y = $2\rho AR^2\omega^2$ (i)

This vertical force is resisted by a force of 2P, such that

$2P = 2\sigma_t A$ (ii)

From equations (i) and (ii), we have

$2\rho AR^2\omega^2 = 2\sigma_t A$

$$\boxed{\sigma_t = \rho R^2\omega^2 = \rho v^2}$$

when ρ is in kg / m^3 and v is in m / s, then σt will be in N / m^2 or Pa.

Note : From the above expression, the mean diameter (D) of the flywheel may be obtained by using the relation,

$$v = \frac{\pi DN}{60}$$

2. Bending stress caused by restraint of the arms

The tensile bending stress in the rim due to the restraint of the arms is based on the assumption that each portion of the rim between a pair of arms behaves like a beam fixed at both ends and uniformly loaded, as shown in Fig. 6, such that length between fixed ends,

$l = \frac{\pi D}{n} = \frac{2\pi R}{n}$, where n = number of arms

The uniformly distributed load (w) per meter length will be equal to the centrifugal force between a pair of arms.

$w = bt\rho\omega^2 R$ N/m

We know that maximum bending moment,

$$M = \frac{w.l^2}{12} = \frac{bt\rho\omega^2 R}{12}\left(\frac{2\pi R}{n}\right)^2$$

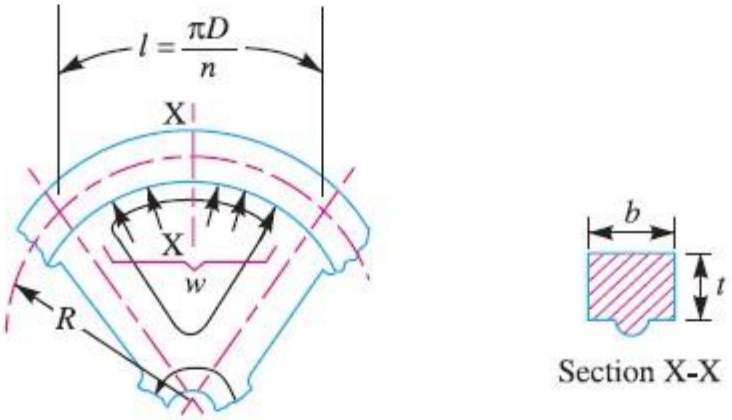


Fig.6 One section of the flywheel rim

And section modulus $Z = \frac{bt^2}{6}$

So bending stress $\sigma_b = \frac{M}{Z} = \frac{bt\rho\omega^2 R}{12}\left(\frac{2\pi R}{n}\right)^2 \times \frac{6}{bt^2} = \frac{19.74\rho\omega^2 R^3}{n^2 t} = \boxed{\frac{19.74\rho v^2 R}{n^2 t}}$

Total stress = $\boxed{\sigma_t + \sigma_b}$

1. **If the arms are placed close to each other and do not stretch, then centrifugal force will not set up stress and** $\sigma_t = 0$
2. **If the arms are stretched , there will be no restrain due to arm and** $\sigma_b = 0$

It has been shown by G. Lanza that arms of a flywheel stretch about 3/4th of the amount necessary for free expansion. Therefore total stress in the rim

$$= \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_b$$

$$= \frac{3}{4}\rho v^2 + \frac{1}{4}\cdot\frac{19.74\rho v^2 R}{n^2 t}$$

$$\boxed{\sigma = \rho v^2\left(0.75 + \frac{4.935R}{n^2 t}\right)}$$

(Eq. 18.6, pp.18.4, Jalaludeen)

Example 2:

A multi-cylinder engine is to run at a constant load at a speed of 600 r.p.m. On drawing the crank effort diagram to a scale of 1 m = 250 N-m and 1 mm = 3º, the areas in sq mm above and below the mean torque line are as follows:

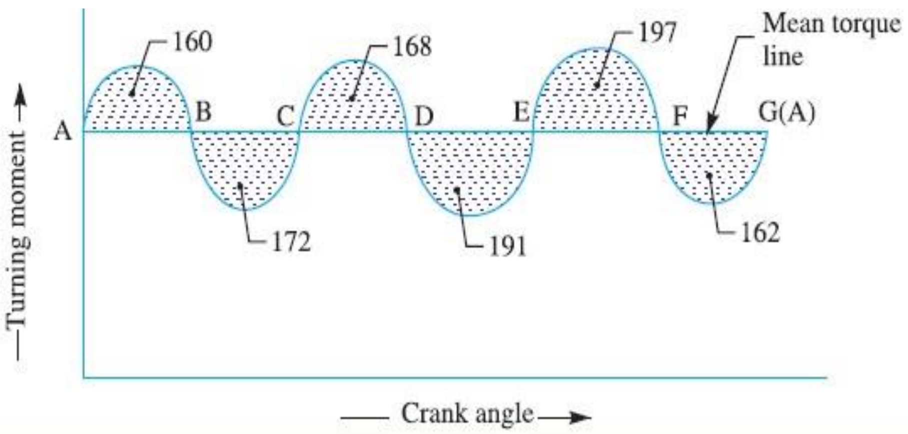
+ 160, – 172, + 168, – 191, + 197, – 162 sq mm

The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel. Determine suitable dimensions for cast iron flywheel with a rim whose breadth is twice its radial thickness. The density of cast iron is 7250 kg / m3, and its working stress in tension is 6 MPa. Assume that the rim contributes 92% of the flywheel effect.

Given : = N = 600 r.p.m. or

$\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; $\rho = 7250$ kg / m^3 ; $\sigma_t = 6$ MPa $= 6 \times 10^6$ N/m^2

Let I = Moment of inertia of the flywheel.



Since the scale for the turning moment is 1 mm = 250 N-m and the scale for the crank angle is $1 \text{ mm} = 3° = \frac{\pi}{60}$ rad, therefore

1 mm^2 on the turning moment diagram

$$= 250 \times \frac{\pi}{60} = 13.1 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.12, we find that

$$\text{Energy at } B = E + 160$$
$$\text{Energy at } C = E + 160 - 172 = E - 12$$
$$\text{Energy at } D = E - 12 + 168 = E + 156$$
$$\text{Energy at } E = E + 156 - 191 = E - 35$$
$$\text{Energy at } F = E - 35 + 197 = E + 162$$

$$\text{Energy at } G = E + 162 - 162 = E = \text{Energy at } A$$

From above, we find that the energy is maximum at F and minimum at E.

$\therefore$ Maximum energy $= E + 162$

and minimum energy $= E - 35$

We know that the maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 162) - (E - 35) = 197 \text{ mm}^2 = 197 \times 13.1 = 2581 \text{ N-m}$$

Since the fluctuation of speed is ± 1% of the mean speed (ω), therefore total fluctuation of speed,

We have $\omega_2 - \omega_1 = 0.02\omega$

The coefficient of fluctuation of speed

$$C_S = \frac{\omega_2 - \omega_1}{\omega} = 0.02$$

We know that the maximum fluctuation of energy (ΔE),

$$2581 = I.\omega^2.C_S = I(62.84)^2\, 0.02 = 79\, I$$ (Eq. 18.1, pp.18.3, Jalaludeen)

$$\therefore \quad I = 2581 / 79 = 32.7 \text{ kg-m}^2$$ **Ans.**

Dimensions of a flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Breadth of the flywheel rim in metres $= 2\,t$ ...(Given)

First of all let us find the peripheral velocity (v) and mean diameter (D) of the flywheel.

We know that tensile stress (σ_t),

We know that tensile stress (σ_t),

$$6 \times 10^6 = \rho.v^2 = 7250 \times v^2$$

$\therefore \quad v^2 = 6 \times 10^6 / 7250 = 827.6 \quad \text{or} \quad v = 28.76 \text{ m/s}$

We also know that peripheral velocity (v),

$$28.76 = \frac{\pi D . N}{60} = \frac{\pi D \times 600}{60} = 31.42 D$$

$\therefore \quad D = 28.76 / 31.42 = 0.915 \text{ m} = 915 \text{ mm}$ **Ans.**

Now let us find the mass of the flywheel rim. Since the rim contributes 92% of the flywheel effect, therefore the energy of the flywheel rim (E_{rim}) will be 0.92 times the total energy of the flywheel (E). We know that maximum fluctuation of energy (ΔE),

$$2581 = E \times 2\, C_S = E \times 2 \times 0.02 = 0.04\, E$$

$\therefore \quad E = 2581 / 0.04 = 64\,525 \text{ N-m}$

and energy of the flywheel rim,

$$E_{rim} = 0.92\, E = 0.92 \times 64\,525 = 59\,363 \text{ N-m}$$

Let $\quad m$ = Mass of the flywheel rim.

We know that energy of the flywheel rim (E_{rim}),

$$59\,363 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m\,(28.76)^2 = 413.6\, m$$

$\therefore \quad m = 59\,363 / 413.6 = 143.5 \text{ kg}$

We also know that mass of the flywheel rim (m),

$$143.5 = b \times t \times \pi D \times \rho = 2\, t \times t \times \pi \times 0.915 \times 7250 = 41\,686\, t^2$$

$\therefore \quad t^2 = 143.5 / 41\,686 = 0.003\,44$

or $\quad t = 0.0587$ say $0.06 \text{ m} = 60 \text{ mm}$ **Ans.**

and $\quad b = 2\,t = 2 \times 60 = 120 \text{ mm}$ **Ans.**

Notes: The mass of the flywheel rim may also be obtained by using the following relations. Since the rim contributes 92% of the flywheel effect, therefore using

1. $\quad I_{rim} = 0.92\, I_{flywheel} \quad \text{or} \quad m.k^2 = 0.92 \times 32.7 = 30 \text{ kg–m}^2$

Since radius of gyration, $k = R = D/2 = 0.915 / 2 = 0.4575$ m, therefore

$$m = \frac{30}{k^2} = \frac{30}{(0.4575)^2} = \frac{30}{0.209} = 143.5 \text{ kg}$$

2.

$$(\Delta E)_{rim} = 0.92\,(\Delta E)_{flywheel}$$

$$m.v^2.C_S = 0.92\,(\Delta E)_{flywheel}$$

$$m\,(28.76)^2\, 0.02 = 0.92 \times 2581$$

$$16.55\, m = 2374.5 \quad \text{or} \quad m = 2374.5 / 16.55 = 143.5 \text{ kg}$$

STRESSES IN FLYWHEEL ARMS

The following stresses are induced in the arms of a flywheel.

1. Tensile stress due to centrifugal force acting on the rim.

2. Bending stress due to the torque transmitted from the rim to the shaft or from the shaft to the rim.

3. Shrinkage stresses due to unequal rate of cooling of casting. These stresses are difficult to determine.

1. Tensile stress due to the centrifugal force

Due to the centrifugal force acting on the rim, the arms will be subjected to direct tensile stress whose magnitude is same as discussed in the previous article.

∴ Tensile stress in the arms, $\sigma_{t1} = \frac{3}{4}\sigma_t = \frac{3}{4}\rho v^2$

2. Bending stress due to the torque transmitted

Due to the torque transmitted from the rim to the shaft or from the shaft to the rim, the arms will be subjected to bending, because they are required to carry the full torque load. In order to find out the maximum bending moment on the arms, it may be assumed as a centilever beam fixed at the hub and carrying a concentrated load at the free end of the rim as shown in Fig. 7.

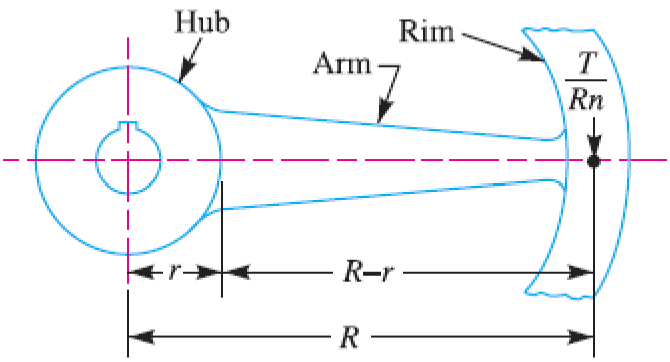


Fig.7 Arm cross-section of the rim

Let

T = Maximum

torque transmitted by the shaft,

R = Mean radius of the rim,

r = Radius of the hub,

n = Number of arms, and

Z = Section modulus for the cross-section of arms.

We know that the load at the mean radius of the rim,

$$F = \frac{T}{R}$$

Load on each arm = $\frac{T}{R.n}$

and maximum bending moment which lies on the arm at the hub,

$$= \frac{T}{R.n}(R - r)$$

So bending stress induced in the flywheel arm $\sigma_{b1} = \frac{M}{Z} = \frac{T}{R.n.Z}(R-r)$

Total tensile stress in the arms at the hub end, $\boxed{\sigma_a = \frac{3}{4}\rho v^2 + \frac{T}{R.n.Z}(R-r)}$ **(pp.18.4, Jalaludeen)**

DESIGN OF FLYWHEEL ARMS

The cross-section of the arms is usually elliptical with major axis as twice the minor axis, as shown in Fig. 8, and it is designed for the maximum bending stress.

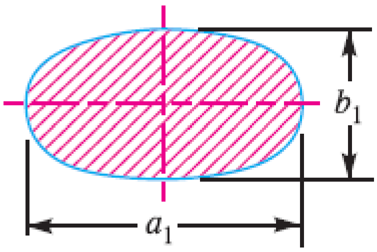


Fig. 8 Cross-section of flywheel arm

Let

a_1 = Major axis, and

b_1 = Minor axis.

Section modulus $\boxed{Z = \frac{\pi}{32} b_1 a_1^2}$ (i)

And maximum bending moment, $M = \frac{T}{R.n}(R-r)$

Maximum bending stress $\boxed{\sigma_{b1} = \frac{M}{Z} = \frac{T}{R.n.Z}(R-r)}$ (ii)

Assuming $a_1 = 2\,b_1$, the dimensions of the arms may be obtained from equations (i) and (ii).

DESIGN OF SHAFT, HUB AND KEY

The diameter of shaft for flywheel is obtained from the maximum torque transmitted. We know that the maximum torque transmitted,

$$\boxed{T_{max} = \frac{\pi}{16} \tau . d_1^3}$$

where

d_1 = Diameter of the shaft, and

τ = Allowable shear stress for the material of the shaft.

The hub is designed as a hollow shaft, for the maximum torque transmitted. We know that the maximum torque transmitted,

$$\boxed{T_{max} = \frac{\pi}{16}\tau.\left(\frac{d^4 - d_1^4}{d}\right)}$$

where

d = Outer diameter of hub, and

d_1 = Inner diameter of hub or diameter of shaft.

The diameter of hub is usually taken as twice the diameter of shaft and length from 2 to 2.5 times the shaft diameter. It is generally taken equal to width of the rim. A standard sunk key is used for the shaft and hub. The length of key is obtained by considering the failure of key in shearing. We know that torque transmitted by shaft,

$$\boxed{T_{max} = L.w.\tau.\frac{d_1}{2}}$$

where

L = Length of the key,

τ = Shear stress for the key material, and

d_1 = Diameter of shaft.

Assignment

Design and draw a cast iron flywheel used for a four stroke I.C engine developing 180 kW at 240 r.p.m. The hoop or centrifugal stress developed in the flywheel is 5.2 MPa, the total fluctuation of speed is to be limited to 3% of the mean speed. The work done during the power stroke is 1/3 more than the average work done during the whole cycle. The maximum torque on the shaft is twice the mean torque. The density of cast iron is 7220 kg/m^3.

Example 3

A punching machine makes 25 working strokes per minute and is capable of punching 25 mm diameter holes in 18 mm thick steel plates having an ultimate shear strength of 300 MPa. The punching operation takes place during 1/10 th of a revolution of the crank shaft. Estimate the power needed for the driving motor, assuming a mechanical efficiency of 95 per cent. Determine suitable dimensions for the rim cross-section of the flywheel, which is to revolve at 9 times the speed of the crank shaft. The permissible coefficient of fluctuation of speed is 0.1.

Given : n = 25 ; d_1 = 25 mm ; t_1 = 18 mm ; τu = 300 MPa = 300 N/mm2 ;

ηm = 95% = 0.95 ; CS = 0.1 ; σ_t = 6 MPa = 6 N/mm2 ; ρ = 7250 kg/m3 ; D = 1.4 m or R = 0.7m

We know that the area of plate sheared,

$$A_S = \pi\, d_1 \times t_1 = \pi \times 25 \times 18 = 1414 \text{ mm}^2$$

∴ Maximum shearing force required for punching,

$$F_S = A_S \times \tau_u = 1414 \times 300 = 424\ 200 \text{ N}$$

and energy required per stroke

$$= {}^*\text{Average shear force} \times \text{Thickness of plate}$$

$$= \frac{1}{2} F_S \times t_1 = \frac{1}{2} \times 424\ 200 \times 18 = 3817.8 \times 10^3 \text{ N-mm}$$

∴ Energy required per min

$$= \text{Energy / stroke} \times \text{No. of working strokes / min}$$

$$= 3817.8 \times 10^3 \times 25 = 95.45 \times 10^6 \text{ N-mm} = 95\ 450 \text{ N-m}$$

We know that the power needed for the driving motor

$$= \frac{\text{Energy required per min}}{60 \times \eta_m} = \frac{95\ 450}{60 \times 0.95} = 1675 \text{ W}$$

$$= 1.675 \text{ kW}$$ **Ans.**

Dimensions for the rim cross-section

Considering the cross-section of the rim as rectangular and assuming the width of rim equal to twice the thickness of rim.

Let t = Thickness of rim in metres, and

b = Width of rim in metres = $2\,t$.

∴ Cross-sectional area of rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

Since the punching operation takes place (*i.e.* energy is consumed) during 1/10 th of a revolution of the crank shaft, therefore during 9/10 th of the revolution of a crank shaft, the energy is stored in the flywheel.

∴ Maximum fluctuation of energy,

$$\Delta E = \frac{9}{10} \times \text{Energy/stroke} = \frac{9}{10} \times 3817.8 \times 10^3$$

$$= 3436 \times 10^3 \text{ N-mm} = 3436 \text{ N-m}$$

Let m = Mass of the flywheel.

Since the hub and the spokes provide 5% of the rotational inertia of the wheel, therefore the maximum fluctuation of energy provided by the flywheel rim will be 95%.

∴ Maximum fluctuation of energy provided by the rim,

$$(\Delta E)_{rim} = 0.95 \times \Delta E = 0.95 \times 3436 = 3264 \text{ N-m}$$

Since the flywheel is to revolve at 9 times the speed of the crankshaft and there are 25 working strokes per minute, therefore mean speed of the flywheel,

$$N = 9 \times 25 = 225 \text{ r.p.m.}$$

and mean angular speed, $\omega = 2\,\pi \times 225 / 60 = 23.56$ rad/s

We know that maximum fluctuation of energy (ΔE),

$$3264 = m.R^2.\omega^2.C_S = m\,(0.7)^2\,(23.56)^2\,0.1 = 27.2\,m$$

∴ $m = 3264 / 27.2 = 120$ kg

We also know that mass of the flywheel (m),

$$120 = A \times \pi\, D \times \rho = 2\,t^2 \times \pi \times 1.4 \times 7250 = 63\ 782\,t^2$$

∴ $t^2 = 120 / 63\ 782 = 0.001\ 88$ or $t = 0.044 \text{ m} = 44 \text{ mm}$ **Ans.**

and $b = 2\,t = 2 \times 44 = 88$ mm **Ans.**

Check for centrifugal stress

We know that peripheral velocity of the rim,

$$v = \frac{\pi\, D\,.\, N}{60} = \frac{\pi \times 1.4 \times 225}{60} = 16.5 \text{ m/s}$$

$\therefore$ Centrifugal stress induced in the rim,

$$\sigma_t = \rho . v^2 = 7250\,(16.5)^2 = 1.97 \times 10^6 \text{ N/m}^2 = 1.97 \text{ MPa}$$

Since the centrifugal stress induced in the rim is less than the permissible value (*i.e.* 6 MPa), therefore it is safe **Ans.**

Module III:

DESIGN OF CLUTCHES

Introduction:

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft.

Types of Clutches:

Following are the two main types of clutches commonly used in engineering practice:

1. Positive clutch,

2. Friction clutch

1. Positive clutch:

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a jaw or claw clutch. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. It consists of two halves, one of which is permanently fastened to the driving shaft by a sunk key. The other half of the clutch is movable and it is free to slide axially on the driven shaft, but it is prevented from turning relatively to its shaft by means of feather key. The jaws of the clutch may be of square type as shown in Fig. 1 (a) or of spiral type as shown in Fig. 1 (b). A square jaw type is used where engagement and disengagement in motion and under load is not necessary. This type of clutch will transmit power in either direction of rotation. The spiral jaws may be left-hand or right-hand, because power transmitted by them is in one direction only. This type of clutch is occasionally used where the clutch must be engaged and disengaged while in motion. The use of jaw clutches are frequently applied to sprocket wheels, gears and pulleys. In such a case, the non-sliding part is made integral with the hub.

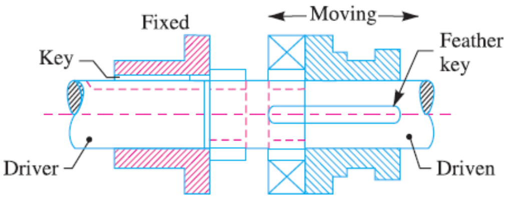


Fig. 1 (a) Square jaw chuck

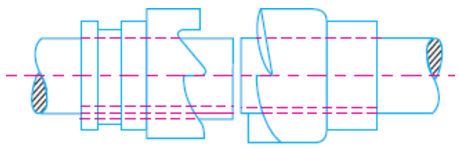


Fig. 1 (b) Spiral jaw chuck

2. Friction Clutch:

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the drive shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually bring the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that :

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
2. The heat of friction should be rapidly *dissipated and tendency to grab should be at a minimum.
3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

Properties of friction material surface:

1. It should have a high and uniform coefficient of friction.
2. It should not be affected by moisture and oil.
3. It should have the ability to withstand high temperatures caused by slippage.
4. It should have high heat conductivity.
5. It should have high resistance to wear and scoring.

Types of Friction Clutches

1. Disc or plate clutches (single disc or multiple disc clutch),
2. Cone clutches, and
3. Centrifugal clutches.

Design of a Disc or Plate Clutch:

Let,

T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

R_o and R_i = External and internal radii of friction face respectively

R_m = Mean radius of the friction face, and

μ= Coefficient of friction. F_a

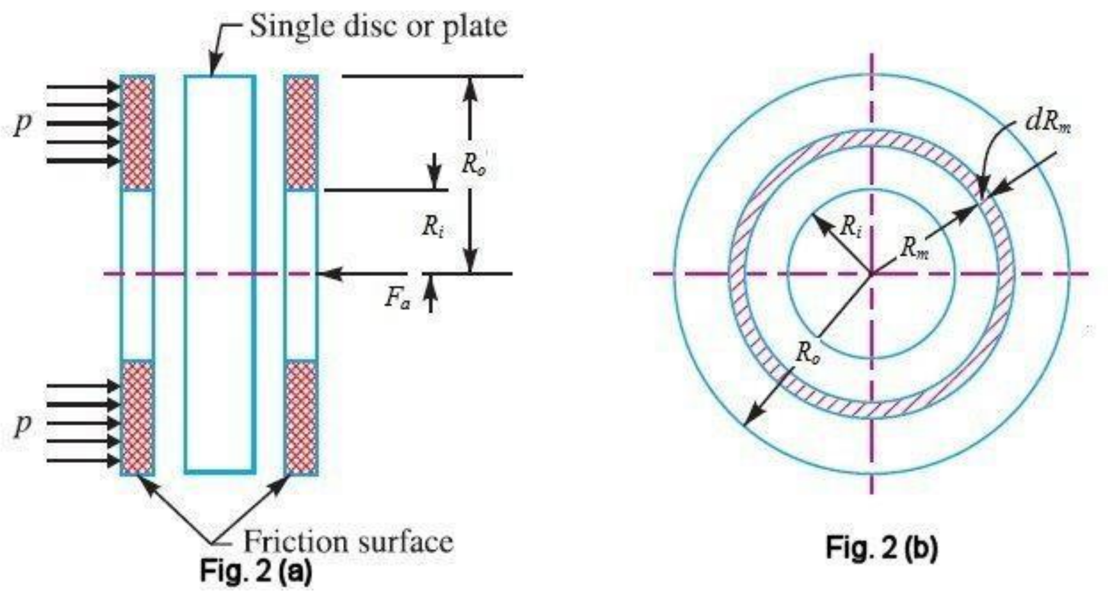


Fig. 2 Forces on the disc

Frictional torque acting on the ring =

The following cases may be considered:

1. When there is a uniform pressure, and
2. When there is a uniform axial wear.

1. Considering uniform pressure:

When the pressure is uniformly distributed over the entire area of the friction face as shown in Fig. 2 (a), then the intensity of pressure,

—— (Eq. 21.8, pp. 21.6, Jalaludeen)

where F_a= Axial thrust with which the friction surfaces are held together

And total designed frictional torque acting on the friction surface or on the clutch,

– [——] (Eq. 21.5, pp. 21.5, Jalaludeen)

Transmitted torque

$\boxed{T = \frac{60 \times 10^6 \times P}{2\pi N} N - mm}$ where P in kW and N in rpm (Eq. 21.1, pp. 21.5, Jalaludeen)

And $\boxed{T_d = T \times k_s}$ (Eq. 21.2, pp. 21.5, Jalaludeen)

Where $\boxed{k_s = k_1 + k_2 + k_3 + k_4}$ (Service factor based on working condition)

(Refer Table 21.2 to 21.5, Jalaludeen)

2. Considering uniform axial wear

The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (p) and the sliding velocity (V).

Total maximum pressure $\boxed{p_{max} = \dfrac{F_a}{2\pi R_i (R_o - R_i)}}$ (Eq. 21.8, pp. 21.6, Jalaludeen)

Total designed frictional torque acting on the friction surface (or on the clutch),

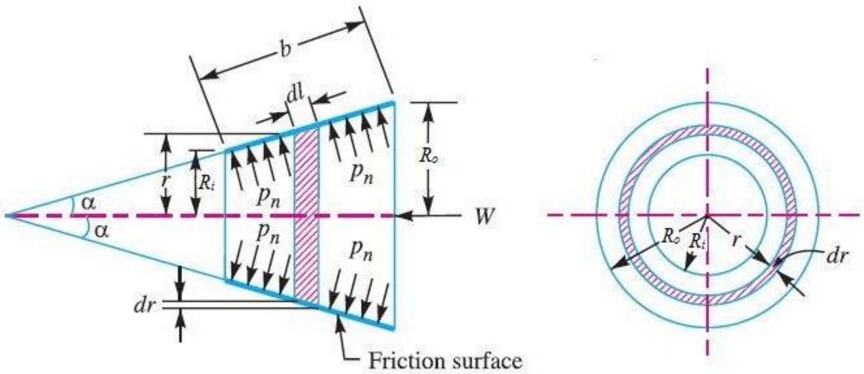
$\boxed{T_d = n\mu F_a R_m}$ (Eq. 21.5, pp. 21.6, Jalaludeen)

Where n = number of pairs of friction surfaces on the clutch

R_m = Mean radius of friction = $\dfrac{R_i + R_o}{2}$ (Eq. 21.6, pp. 21.6, Jalaludeen)

Design of Cone Clutch

Consider a pair of friction surfaces of a cone clutch as shown in Figure. A little consideration will show that the area of contact of a pair of friction surface is a frustrum of a cone.



Cone Clutch details

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (i.e. normal pressure between the contact surfaces),

α = Semi-angle of the cone (also called face angle of the cone) or angle of the friction surface with the axis of the clutch = 12.5^0 to 25^0

μ = Coefficient of friction between the contact surfaces, and

b = Width of the friction surfaces (also known as face width or cone face).

R_o = Outer radius of clutch plate = $\boxed{R_m + \dfrac{b}{2}\sin\alpha}$ or $\boxed{R_i + b\sin\alpha}$ (Eq. 21.10, pp. 21.7, Jalaludeen)

R_i = Inner radius of clutch plate = $\boxed{R_m - \dfrac{b}{2}\sin\alpha}$ (Eq. 21.10, pp. 21.7, Jalaludeen)

And R_m = (2.5 to 5) d or (2 to 4) b (Eq. 21.10, pp. 21.7, Jalaludeen)

d = Dia. Of clutch shaft

1. **Considering Uniform Pressure**

Total axial load transmitted to the clutch or the axial spring force required, $\boxed{F_a = \pi p_n \left[R_o^2 - R_i^2\right]}$

Total designed torque $T_d = \mu R_m F_a \operatorname{cosec}\alpha$, and $R_m = \dfrac{R_i + R_o}{2}$ (Eq. 21.12, pp. 21.7, Jalaludeen)

Module IV **DESIGN OF JOURNAL BEARING**

1. Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig. 1, in which O is the centre of the journal and O′ is the centre of the bearing.

Let, D = Diameter of the bearing

d = diameter of journal

l= length of bearing

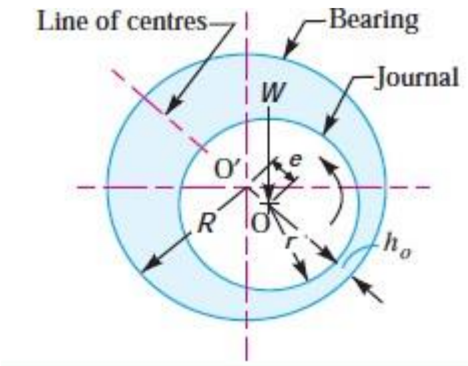


Fig. 1: Hydrodynamic Journal Bearing

The following terms used in hydrodynamic journal bearing are important from the subject point of view.

1. **Diametral clearance**. It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance, $\boxed{c = D - d}$.
2. **Radial clearance**. It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance, $\boxed{c_1 = R - r = \frac{D-d}{2} = \frac{c}{2}}$.
3. **Diametral clearance ratio**. It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio, $\boxed{\frac{c}{d} = \frac{D-d}{d}}$.
4. **Eccentricity**. It is the radial distance between the centre (O) of the bearing and the displaced centre (O′) of the bearing under load. It is denoted by e.
5. **Minimum oil film thickness**. It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h0 and occurs at the line of centres as shown in Fig. 1. Its value may be assumed as c / 4.

6. **Attitude or eccentricity ratio**. It is the ratio of the eccentricity to the radial clearance.

Mathematically, attitude or eccentricity ratio, $\boxed{\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_o}{c_1} = 1 - \frac{h_o}{c_1} = 1 - \frac{2h_o}{c}}$.

7. **Short and long bearing**. If the ratio of the length to the diameter of the journal (i.e. l / d) is less than 1, then the bearing is said to be short bearing. On the other hand, if l / d is greater than 1, then the bearing is known as long bearing.

Notes : 1. When the length of the journal (l) is equal to the diameter of the journal (d), then the bearing is called square bearing.

2. Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, i.e. $\frac{Zn}{p}, \frac{d}{c}, \frac{l}{d}$.

Therefore the coefficient of friction may be expressed as $\boxed{\mu = \phi\left(\frac{Zn}{p}, \frac{d}{c}, \frac{l}{d}\right)}$

Where ,

μ = Coefficient of friction,

ϕ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

n = Speed of the journal in r.p.m.,

p = Bearing pressure on the projected bearing area in N/mm^2, = Load on the journal ÷ l × d

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor ZN / p is termed as bearing characteristic number and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. 2. The factor ZN/p helps to predict the performance of a bearing.

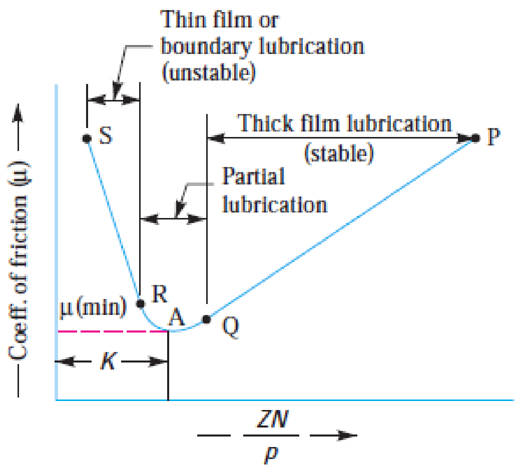


Fig. 2 Variation of coefficient of friction with ZN/p.

The part of the curve PQ represents the region of thick film lubrication. Between Q and R, the viscosity (Z) or the speed (N) are so low, or the pressure (p) is so great that their combination ZN / p will reduce the film thickness so that partial metal to metal contact will result. The thin film or boundary lubrication or imperfect lubrication exists between R and S on the curve. This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete metal to metal contact and seizure of the parts. It may be noted that the part PQ of the curve represents stable operating conditions, since from any point of stability, a decrease in viscosity (Z) will reduce Zn / p. This will result in a decrease in coefficient of friction (μ) followed by a lowering of bearing temperature that will raise the viscosity (Z). From Fig. 2, we see that the minimum amount of friction occurs at A and at this point the value of Zn / p is known as bearing modulus which is denoted by K. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of Zn / p at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of Zn / p = 15 K may be used. From above, it is concluded that when the value of ZN / p is greater than K, then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of ZN / p is less than K, then the oil film will rupture and there is a metal to metal contact.

3. <u>Coefficient of Friction for Journal Bearings</u>

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

Coefficient of friction $\boxed{\mu = \left[\frac{33.25}{10^8} \times \frac{Zn}{p} \times \frac{d}{c}\right] + k}$ (Eq. 19.5, pp. 19.3, Jalaludeen)

(When Z in N-s/m^2, or kg/m-s and p in N/mm^2)

$\boxed{\mu = \left[\frac{33.25}{10^{10}} \times \frac{Zn}{p} \times \frac{d}{c}\right] + k}$ (Eq. 19.6, pp. 19.3, Jalaludeen)

(When Z in centipoise, or kg/m-s and p in kgf/cm^2)

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (i.e. l / d). (Refer Fig. 19.2, pp.19.24, Jalaludeen) , and The design values can be taken from Table 19.5, pp. 19.13, Jalaludeen.

4. **Critical Pressure of the Journal Bearing**

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following empirical relation, i.e. Critical pressure or minimum operating

pressure, $\boxed{p_c = \frac{Zn}{4.75\times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{l+d}\right)}$ N/mm^2 (Eq. 19.15, pp. 19.6, Jalaludeen)

When, Z in N-s/m^2

And $\boxed{p_c = \frac{Zn}{475\times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{l+d}\right)}$ kgf.cm^2 (Eq. 19.16, pp. 19.6, Jalaludeen), When, Z

centipoise

5. **Sommerfeld Number**

The Sommerfeld number is also a dimensionless parameter used extensively in the design of

journal bearings. Mathematically, $\boxed{S = \frac{Zn}{60\times 10^6 p}\left(\frac{d}{c}\right)^2}$ (Eq. 19.8, pp. 19.4, Jalaludeen),

Table 19.7 to Table 19.10.

6. **Heat Generated in a Journal Bearing**

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing, $\boxed{H_g = \mu WV}$ watts.

(when the load on bearing W in Newtons and V in m/s)

And $\boxed{H_g = \mu WV}$ kgf-m/min or $\dfrac{\mu WV}{J}$ kcal/min (Eq. 19.10, pp. 19.4, Jalaludeen)

(when W in Newtons and the rubbing velocity V in m/s)

And $\boxed{V = \dfrac{\pi dn}{60}}$ m/s

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing $\boxed{H_d = CA(t_b - t_a)}$ (Eq. 19.12, pp. 19.5, Jalaludeen)

Where C= Heat dissipation coefficient (values can be obtained from pp. 19.5, Jalaludeen)

A = projected area = $l \times d$ in m^2

t_b = temp. of bearing in ^oC

t_a = temp. of surroundings in ^oC

7. <u>Design Procedure for Journal Bearing</u>

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

1 Determine the bearing length by choosing a ratio of l / d from Table 19.5, pp. 19.13, Jalaludeen.

2 Check the bearing pressure, p = W / l.d (Eq. 19.9, pp. 19.4, Jalaludeen), from Table 19.5, pp. pp. 19.13, Jalaludeen, for probable satisfactory value.

3 Assume a lubricant from Table 19.11, pp. 19.26, Jalaludeen, and its operating temperature (t0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.

4 Determine the operating value of Zn / p for the assumed bearing temperature and check this value with corresponding values in Table 19.5, pp. 19.13, Jalaludeen to determine the possibility of maintaining fluid film operation.

5 Assume a clearance ratio c / d from Table 19.5, pp. 19.13, Jalaludeen.

6 Determine the coefficient of friction (μ) by using the relation as discussed in Art. 3.

7 Determine the heat generated by using the relation as discussed in Art. 6.

8 Determine the heat dissipated by using the relation as discussed in Art. 6.

9 Determine the thermal equilibrium to see that the heat dissipated becomes at least equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

BALL AND ROLLER BEARING

Advantages and disadvantages of Roller bearing over sliding bearing

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

Types of Rolling Contact Bearings

Following are the two types of rolling contact bearings:

1. Ball bearings; and
2. Roller bearings.

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig. 3. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads. The rolling contact bearings, depending upon the load to be carried, are classified as : (a) Radial bearings, and (b) Thrust bearings.

The radial and thrust ball bearings are shown in Fig. 4 (a) and (b) respectively. When a ball bearing supports only a radial load (WR), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. 4 (a). The action of thrust load (WA) is to shift the plane of rotation of the balls, as shown in Fig. 4 (b). The radial and thrust loads both may be carried simultaneously.

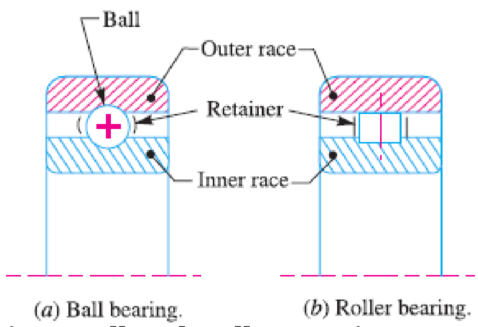


Fig. 3 Ball and Roller Bearing

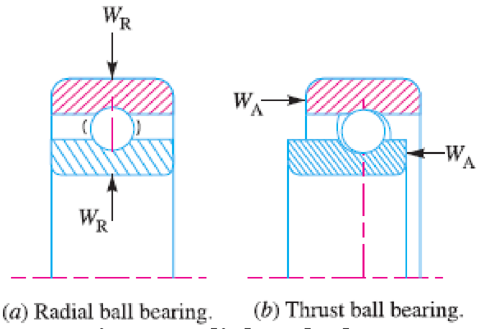


Fig. 4 Radial and Thrust Bearing

Types of Radial Ball Bearings

Following are the various types of radial ball bearings:

1. <u>Single row deep groove bearing. A single row deep groove bearing as shown in Fig. 5 (a).</u>

During assembly of this bearing, the races are offset and the maximum numbers of balls are placed between the races. The races are then centered and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.

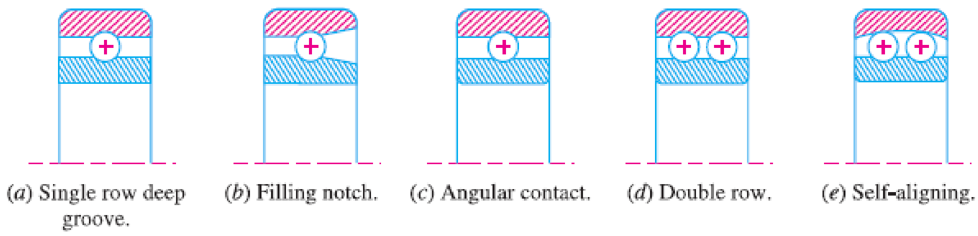


Fig. 5 Types of Radial Ball Bearing

2. <u>Filling notch bearing.</u>

A filling notch bearing is shown in Fig. 5 (b). These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearing. The notches do not extend to the bottom of the race way and therefore the balls inserted through the notches must be forced in position. Since this type of bearing contains larger number of balls than a corresponding un-notched one, therefore it has a larger bearing load capacity.

3. Angular contact bearing.

An angular contact bearing is shown in Fig. 5 (c). These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. Double row bearing.

A double row bearing is shown in Fig. 5 (d). These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. Self-aligning bearing.

A self-aligning bearing is shown in Fig. 5 (e). These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of self-aligning bearings:

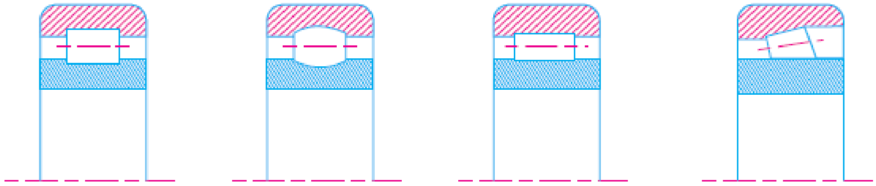
(a) Externally self-aligning bearing, and (b) Internally self-aligning bearing.

In an externally self-aligning bearing, the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in a housing, as shown in Fig. 5 (e). In case of internally self-aligning bearing, the inner surface of the outer race is ground to a spherical surface. Consequently, the outer race may be displaced through a small angle without interfering with the normal operation of the bearing. The internally self-aligning ball bearing is interchangeable with other ball bearings.

Types of Roller Bearings

Following are the principal types of roller bearings :

1. Cylindrical roller bearings. A cylindrical roller bearing is shown in Fig. 6(a). These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such types of bearings are used in high speed service.



(a) Cylindrical roller. (b) Spherical roller. (c) Needle roller. (d) Tapered roller.

Fig. 6 Types of Roller Bearing

2. Spherical roller bearings. A spherical roller bearing is shown in Fig. 6 (b). These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere. These bearings can normally tolerate angular misalignment in the order of $\pm 1\frac{1}{2}^{\circ}$ and when used with a double row of rollers, these can carry thrust loads in either direction.

3. Needle roller bearings. A needle roller bearing is shown in Fig. 6 (c). These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, e.g. piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

4. Tapered roller bearings. A tapered roller bearing is shown in Fig. 6 (d). The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.

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